

§ 2.1

2 – 8 (even), 14 – 28 (even), 32, 34, 36, 44, 50, 52, 58, 62, 66, 70

Solutions

1 – 4 Express the rule in function notation

2) Divide by 7, then subtract 4

$$f(x) = \frac{x}{7} - 4$$

4) Take the square root, add 8, then multiply by $\frac{1}{3}$

$$f(x) = \frac{1}{3}(\sqrt{x} + 8)$$

5 – 8 Express the function in words

6) $g(x) = \frac{x}{3} - 4$

Divide by 3, then subtract 4

8) $k(x) = \sqrt{x + 2}$

Add 2, then take the square root

13 – 20 Evaluate the function at the indicated values

14) $f(x) = x^2 + 2x$

$$f(0) = 0^2 + 2(0) = 0$$

$$f(3) = 3^2 + 2(3) = 15$$

$$f(a) = a^2 + 2a$$

$$f(-x) = (-x)^2 + 2(-x) = x^2 - 2x$$

$$f\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)^2 + 2\left(\frac{1}{a}\right) = \frac{1}{a^2} + \frac{2}{a} = \frac{1 + 2a}{a^2}$$

Math 1650 Homework Solutions

Jason Snyder, PhD.

$$16) h(t) = t + \frac{1}{t}$$

$$h(1) = 1 + \frac{1}{1} = 2$$

$$h(-1) = -1 + \frac{1}{-1} = -1 - 1 = -2$$

$$h(2) = 2 + \frac{1}{2} = \frac{5}{2}$$

$$h\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{1/2} = \frac{1}{2} + 2 = \frac{5}{2}$$

$$h(x) = x + \frac{1}{x}$$

$$h\left(\frac{1}{x}\right) = \frac{1}{x} + \frac{1}{1/x} = \frac{1}{x} + x = h(x)$$

$$18) f(x) = x^3 - 4x^2$$

$$f(0) = 0^3 - 4(0)^2 = 0$$

$$f(1) = 1^3 - 4(1)^2 = -3$$

$$f(-1) = (-1)^3 - 4(-1)^2 = -1 - 4 = -5$$

$$f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^3 - 4\left(\frac{3}{2}\right)^2 = \frac{27}{8} - 9 = \frac{27}{8} - \frac{72}{8} = -\frac{45}{8}$$

$$f\left(\frac{x}{2}\right) = \left(\frac{x}{2}\right)^3 - 4\left(\frac{x}{2}\right)^2 = \frac{x^3}{8} - x^2 = \frac{x^3}{8} - \frac{8x^2}{8} = \frac{x^2(x-8)}{8}$$

$$f(x^2) = (x^2)^3 - 4(x^2)^2 = x^6 - 4x^4 = x^4(x^2 - 4) = x^4(x+2)(x-2)$$

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$$20) f(x) = \frac{|x|}{x}$$

$$f(-2) = \frac{|-2|}{-2} = \frac{2}{-2} = -1$$

$$f(-1) = \frac{|-1|}{-1} = \frac{1}{-1} = -1$$

$f(0)$ is undefined

$$f(5) = \frac{|5|}{5} = \frac{5}{5} = 1$$

$$f(x^2) = \frac{|x^2|}{x^2} = \frac{x^2}{x^2} = 1$$

$$f\left(\frac{1}{2}\right) = \frac{\left|\frac{1}{x}\right|}{\frac{1}{x}} = \frac{1/|x|}{1/x} = \frac{x}{|x|}$$

21 – 24 Evaluate the piecewise defined function at the indicated values

22)

$$f(x) = \begin{cases} 5 & x \leq 2 \\ 2x - 3 & x > 2 \end{cases}$$

$$f(-3) = 5$$

$$f(0) = 5$$

$$f(2) = 5$$

$$f(3) = 2(3) - 3 = 3$$

$$f(5) = 2(5) - 3 = 7$$

24)

$$f(x) = \begin{cases} 3x & x < 0 \\ x + 1 & 0 \leq x \leq 2 \\ (x - 2)^2 & x > 2 \end{cases}$$

$$f(-5) = 3(-5) = -15$$

$$f(0) = 0 + 1 = 1$$

$$f(1) = 1 + 1 = 2$$

$$f(2) = 2 + 1 = 3$$

$$f(5) = (5 - 2)^2 = 3^2 = 9$$

25 – 28 Use the function to evaluate the indicated expressions and simplify

26) $f(x) = 3x - 1$

$$f(2x) = 3(2x) - 1 = 6x - 1$$

$$2f(x) = 2(3x - 1) = 6x - 2$$

28) $f(x) = 6x - 18$

$$f\left(\frac{x}{3}\right) = 6\left(\frac{x}{3}\right) - 18 = 2x - 18$$

$$\frac{f(x)}{3} = \frac{6x - 18}{3} = 2x - 6$$

29 – 36 Find $f(a)$, $f(a + h)$, and the difference quotient $\frac{f(a+h)-f(a)}{h}$, where $h \neq 0$.

32) $f(x) = x^2 + 1$

$$f(a) = a^2 + 1$$

$$f(a + h) = (a + h)^2 + 1 = a^2 + 2ah + h^2 + 1$$

$$\frac{f(a + h) - f(a)}{h} = \frac{a^2 + 2ah + h^2 + 1 - a^2 - 1}{h} = \frac{2ah + h^2}{h} = 2a + h$$

$$34) f(x) = \frac{2x}{x-1}$$

$$f(a) = \frac{2a}{a-1}$$

$$f(a+h) = \frac{2(a+h)}{a+h-1} = \frac{2a+2h}{a+h-1}$$

$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{\frac{2a+2h}{a+h-1} - \frac{2a}{a-1}}{h} = \frac{\frac{-2ah}{(a+h-1)(a-1)}}{h} \\ &= -\frac{2a}{(a+h-1)(a-1)} \end{aligned}$$

$$36) f(x) = x^3$$

$$f(a) = a^3$$

$$f(a+h) = (a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$$

$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a^3}{h} = \frac{3a^2h + 3ah^2 + h^3}{h} \\ &= 3a^2 + 3ah + h^2 \end{aligned}$$

37 – 58 Find the domain of the function

$$44) f(x) = \frac{x^4}{x^2+x-6} = \frac{x^4}{(x+3)(x-2)}$$

$$\text{Domain of } f = \{x : x \neq -3 \text{ and } x \neq 2\}$$

$$50) G(x) = \sqrt{x^2 - 9}$$

$$x^2 - 9 \geq 0$$

$$x^2 \geq 9$$

$$x \leq -3 \text{ or } x \geq 3$$

$$\text{Domain of } G = \{x : x \leq -3 \text{ or } x \geq 3\}$$

$$52) g(x) = \frac{\sqrt{x}}{2x^2+x-1} = \frac{\sqrt{x}}{(2x-1)(x+1)}$$

$$\text{Domain of } g = \left\{x \geq 0 : x \neq \frac{1}{2}\right\}$$

$$58) f(x) = \frac{x}{\sqrt{9-x^2}}$$

$$9 - x^2 \geq 0$$

$$x^2 \leq 9$$

$$-3 \leq x \leq 3$$

$$\text{Domain of } f = \{x : -3 \leq x \leq 3\}$$

62) **Toricelli's Law**

A tank holds 50 gallons of water, which drains from a leak at the bottom, causing the tank to empty in 20 minutes. The tank drains faster when it is nearly full because the pressure on the leak is greater. **Toricelli's Law** gives the volume of water remaining in the tank after t minutes as

$$V(t) = 50 \left(1 - \frac{t}{20}\right)^2 \quad 0 \leq t \leq 20$$

(a) Find $V(0)$ and $V(20)$

$$V(0) = 50 \left(1 - \frac{0}{20}\right)^2 = 50 \text{ gallons}$$

$$V(20) = 50 \left(1 - \frac{20}{20}\right)^2 = 0 \text{ gallons}$$

(b) What do your answers from part (a) represent?

The initial and final volumes of the tank.

(c) Make a table of values of $V(t)$ for $t = 0, 5, 10, 15, 20$.

t	$V(t)$
0	50
5	$225/8$
10	$25/2$
15	$25/8$
20	0

66) **Income Tax**

In a certain country, income tax T is assessed according to the following function of income x :

$$T(x) = \begin{cases} 0 & 0 \leq x \leq 10,000 \\ 0.08x & 10,000 < x \leq 20,000 \\ 1600 + 0.15x & 20,000 < x \end{cases}$$

(a) Find $f(5000)$, $f(12,000)$, and $f(25,000)$

$$f(5,000) = 0$$

$$f(12,000) = 0.08(12,000) = 960$$

$$f(25,000) = 1600 + 0.15(25,000) = 5,350$$

(b) What do your answers from part (a) represent?

The amount of tax owed for \$5,000, \$12,000, and \$25,000 incomes respectfully.

70) **Height of Grass**

A home owner mows a lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four week period beginning on a Sunday.

